Total No. of Questions : 8]

SEAT No. :

P362

[Total No. of Pages : 2

[4223] - 101

M.Sc.

MATHEMATICS

MT-501 : Real Analysis - I

(2008 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) If C $[a, b]$ denote the set of all complex valued continuous function on $[a, b]$, then show that $\langle f, g\rangle = \int_a^b f(x)\overline{g(x)}\,dx$ is an inner product space on C $[a, b]$. **[6]**

b) Consider the norms $\|.\|_1$ and $\|.\|_\infty$ on R^n Prove that $\|x\| = \frac{1}{3}\|x\|_1 + \frac{2}{3}\|x\|_\infty$ defines a norm on R^n. **[5]**

c) Prove that l^1 is infinite - dimensional. Also show that l^∞ is infinite -dimensional. **[5]**

Q2) a) If a subset of a metric space is compact then prove that it is sequentially compact. **[6]**

b) Show that C ($[a, b]$, R) with the supremum norm is complete. **[6]**

c) Justify whether the following statement is true or false int $(A\cup B)$ = int (A) $\cup$ int (B). **[4]**

Q3) a) Assume that μ is a countably additive function defined on a ring R, A, $A_n \in R$ that $A_1 \subseteq A_2 \subseteq$ ---, and $A = \bigcup_{n=1}^{\infty} A_n$

Prove that $\lim_{n\to\infty} \mu(A_n) = \mu(A)$ **[6]**

b) Let m be the lebesgue measure defined on R^n. Let ε be the collection of all finite unions of disjoint intervals in R^n. Prove that m is a measure on ε. **[6]**

c) Define measurable sets in R^n and show that family of measurable set is σ - ring. **[4]**

P.T.O.

Q4) a) Assume that $f \geq 0$ is measurable function and that A_1, A_2 ..... are pairwise disjoint measurable sets. Prove that $\int_{\bigcup_{k=1}^{\infty} A_k} f\,dm = \sum_{k=1}^{\infty} \int_{A_k} f\,dm$ **[7]**

b) For every integrable function f with respect to m, over R^n prove that $\int_E f\,dm = 0$ for every measurable set E of measure zero. Also prove that $\int_A f\,dm = \int_B f\,dm$ Whenever A and B are measurable sets, $B \subseteq A$, and $m(A \setminus B) = 0$. **[6]**

c) If f is measurable then prove that $|f|$ is measurable. **[3]**

Q5) a) State and prove Lebesgue Dominated Convergence Theorem. **[8]**

b) Prove that every continuous real-valued function defined on R^n is measurable. **[5]**

c) For $1 \leq p < \infty$ define $L^p(\mu)$ and prove that it is linear space. **[3]**

Q6) a) State and prove Fatou's Lemma. **[8]**

b) Prove that $L^{\infty}(\mu)$ is complete. **[6]**

c) Define simple function. Give an example of a function which is not measurable. **[2]**

Q7) a) Find the Fourier series of $f(x) = x$. **[6]**

b) Apply Gram - Schmidt process to the functions 1, x, x^2, ... and find formulae for first three Legendre Polynomial. **[5]**

c) i) Prove that in any inner product space $(V, < >)$, f and g are orthogonal implies that $\|f\|^2 + \|g\|^2 = \|f + g\|^2$

ii) Give an example to show that pointwise convergence does not imply uniform convergence. **[5]**

Q8) a) State and prove Bessels Inequality. **[8]**

b) Let X be a complete metric space and T a contraction from X into X show that there exists a unique fixed point of T. **[6]**

c) State Stone-Weierstrass Theorem. **[2]**

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Total No. of Questions : 8]

SEAT No. :

P363

[Total No. of Pages : 3

[4223] - 102

M.Sc.

MATHEMATICS

MT - 502 : Advanced Calculus

(2008 Pattern) (semester - I)

Time : 3 Hours] ***[Max. Marks :80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Define continuity of vector field. Prove that a linear transformation $\vec{f}: \mathrm{R}^n \to \mathrm{R}^m$ is continuous for every $\vec{a} \in \mathrm{R}^n$. **[4]**

b) Define

i) Derivative of a scalar field.

ii) Directional derivative of a scalar field.

Explain and illustrate by an example the difference of the above two definitions. **[8]**

c) Find the gradient vector at each point of the scalar field $f(x, y) = x^2 + y^2 \sin(x y)$, if it exists. **[4]**

Q2) a) Let $f : \mathrm{R}^n \to \mathrm{R}$ be a scalar field and $\vec{a} \in \mathrm{R}^n$. Assume that the partial derivatives $\mathrm{D}_1 f, \ldots\ldots \mathrm{D}_n f$ exist in some n-ball $\mathrm{B}(\vec{a})$ and are continuous at $\vec{a}$. Then prove that f is differentiable at $\vec{a}$. **[8]**

b) Compute $\mathrm{F}'(t)$ and $\mathrm{F}''(t)$ in terms of t, where $u = \mathrm{F}(t)$, $u = f(x, y) = e^{xy} \cos(xy^2)$, $x = x(t) = \cos t$, $y = y(t) = \sin t$. **[5]**

c) Let Z be a function of u and v, where $u = x^2 - y^2 - 2xy$ and $v = y$. Find $(x+y)\dfrac{\partial z}{\partial x} + (x-y)\dfrac{\partial z}{\partial y}$. **[3]**

P.T.O.

Q3) a) State only the implicit function theorem. **[2]**

b) Let $\vec{h} = \vec{f} o \vec{g}$, where $\vec{g}$ is differentiable at $\vec{a}$ and $\vec{f}$ is differentiable at $\vec{b} = \vec{g}(\vec{a})$. Suppose $\vec{a} \in \mathrm{R}^p$, $\vec{b} \in \mathrm{R}^n$ and $\vec{f}(\vec{b}) \in \mathrm{R}^m$, next suppose .

$\vec{g} = (g_1, g_2, ..., g_n)$, $\vec{f} = (f_1, ..., f_m)$, $\vec{h} = (h_1, ... , h_m)$. By using chain rule show that $\mathrm{D}_j h_i(\vec{a}) = \sum_{K=1}^{n} \mathrm{D}_k f_i(\vec{b}) \mathrm{D}_j g_k(\vec{a})$ for i = 1, 2, ..., m and j=1, 2, ... , p. **[6]**

c) Let $\vec{f}: \mathrm{R}^2 \to \mathrm{R}^2$ and $\vec{g}: \mathrm{R}^3 \to \mathrm{R}^2$ be two vector fields defined as follows :

$\vec{f}(x, y) = e^{x+2y}\,\vec{i} + \sin(y+2x)\,\vec{j}$

$\vec{g}(u, v, w) = (u + 2v^2 + 3w^3)\,\vec{i} + (2v - u^2)\,\vec{j}$

verify the chain rule in terms of partial derivatives obtained in (b) above, with $\vec{a} = (1, -1, 1)$. **[8]**

Q4) a) Define line integral and illustrate by an example. **[4]**

b) Prove that the line integral of a gradient is independent of the path in any open connected set in which the gradient is continuous. **[6]**

c) Evaluate $\int_c \frac{dx + dy}{|x| + |y|}$, where C is the square with vertices (1, 0), (0, 1), (–1,0) and (0, –1) traversed once in a counterclockwise direction. **[6]**

Q5) a) Define double integral of a bounded function over a rectangle.

Prove that every function *f* which is bounded on a rectangle Q has a lower integral $\underline{\mathrm{I}}(f)$ and an upper integral $\bar{\mathrm{I}}(f)$, further prove that *f* in integrable over Q if and only if its upper and lower integrals are equal. **[8]**

b) Evaluate $\iint_s e^{x+y}\, dxdy$ where $\mathrm{S} = \{(x, y) / |x| + |y| \le 1\}$. **[5]**

c) Check whether $\vec{f}(x, y) = 3x^2 y\,\vec{i} + x^3 j$ is gradient of a scalar field. **[3]**

Q6) a) In usual notations prove the transformation formula

$$\iint_{\mathrm{S}} f(x, y)\, dxdy = \iint_{\mathrm{T}} f\,[x(u, v),\; y(u, v)]\,|\,\mathrm{J}(u, v)\,|\,dudv$$

where $f(x, y) = 1$ on rectangle S. **[8]**

b) Prove the transformation formula:

$$\iiint\limits_S f(x,y,z)\,dxdydz = \iiint\limits_T F(\rho,\theta,\varphi)\,\rho^2 \sin\varphi\, d\rho\, d\theta\, d\varphi,$$

where $x = \rho\cos\theta\sin\varphi$, $y = \rho\sin\theta\sin\varphi$, $z = \rho\cos\varphi$ and $\rho > 0$, $0 \le \theta < 2\pi$ and $0 \le \varphi < \pi$. Justify your steps. **[8]**

Q7) a) Define fundamental vector product of a parametric surface.

Consider the hemisphere given by

$\vec{\gamma}(u,v) = a\cos u\,\cos v\,\vec{i} + a\sin u\,\cos v\,\vec{j} + a\sin v\,\vec{k}$,

where $T = [0, 2\pi]\times\left[0, \frac{1}{2}\pi\right]$ Find $\frac{\partial\vec{r}}{\partial u}\times\frac{\partial\vec{r}}{\partial v}$. What are singular points of this surface? **[8]**

b) Let S denote the hemisphere $x^2 + y^2 + z^2 = 1$, $z \ge 0$, and let $\vec{F}(x,y,z) = x\vec{i} + y\vec{j}$. Let $\vec{n}$ be the unit outward normal of S. Compute the value of the surface integral $\iint\limits_s \vec{F}.\vec{n}\,ds$, using the explicit representation.

$Z = \sqrt{1-x^2-y^2}$. **[5]**

c) State only the theorem of change of parametric representation of a surface integral. **[3]**

Q8) a) State and prove stokes theorem. **[6]**

b) Let $\vec{F}(x,y,z) = (x^2+yz)\vec{i} + (y^2+xz)\vec{j} + (z^2+xy)\vec{k}$, then find curl and divergence of $\vec{F}$ by computing its Jacobian matrix. **[5]**

c) A double integral of a positive function f, $\iint\limits_s f(x,y)\,dxdy$, reduces to the repeated integral : $\int_0^3\left[\int_{4y/3}^{\sqrt{25-y^2}} f(x,y)\,dx\right]dy$.

Determine the region S and interchange the order of integration. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P367

[Total No. of Pages : 2

[4223] - 201
M.Sc.
MATHEMATICS
MT - 601 : General Topology
(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let A be any non empty set in a topological space X. Suppose for each $x \in$ A, there is an open set U containing x such that U$\subset$A, show that A is open in X. **[4]**

b) Define usual topology an lower limit topology on R. Establish a relation among them. **[6]**

c) If X = $\{a, b, c\}$, let $J_1 = \{\phi, X, \{a\}, \{a, b\}\}$

$J_2 = \{\phi, x, \{a\}, \{b, c\}\}$ find largest topology contained in J_1 and J_2; and smallest topology containing J_1 and J_2. **[6]**

Q2) a) If $\{J_\alpha\}$ is a family of topologies on X. Show that $\cap\ J_\alpha$ is a topology on X. Is $\cup J_\alpha$ a topology on X? Justify **[8]**

b) Show that thecollection $S=\{\pi_1^{-1}(U)/U \text{ open in} X\} \cup \{\pi_2^{-1}(V)/V \text{open in } Y\}$ is a sub basis for the product topology on X × Y. **[6]**

c) Define limit point of a set in a topological space. Find limit points of $A = (0, 1]$ and $B=\left\{\frac{1}{n}/n\in Z_+\right\}$ in usual topology on R. **[2]**

Q3) a) Let (X, J) be a T_1 space and A$\subset$X. Show that x is a limit point of A $\Leftrightarrow$ every neighbourhood of x contains infinitely many points of A. **[6]**

b) State and prove pasting lemma. **[6]**

c) Define Box topology and Product topology. What is relation between them? **[4]**

P.T.O.

Q4) a) Let (X, J) be a topological space and the sets C and D forms a separation of X. If Y is connected subspace of X then show that Y lies entirely within either C or D. **[6]**

b) Prove that arbitrary union of connected subspaces of a topological space, having one point in common is connected. **[5]**

c) Let f = (X, J) $\rightarrow$ (Y, J') be a continuous map. Show that if there is a continuous map g : Y $\rightarrow$ X such that fog equals the identity map of Y then f is a quotient map. **[5]**

Q5) a) Show that compact subspace of a Hausdorff space is closed. **[6]**

b) Show that continuous image of compact set is compact. **[5]**

c) Prove that compactness implies limit point compactness. Is converse true? Justify. **[5]**

Q6) a) Show that finite product of compact spaces is compact. **[8]**

b) Define the following terms with examples : **[6]**

i) Limit point compactness.

ii) Local compactness.

c) Show that not every first countable space is second countable. **[2]**

Q7) a) State and prove Tychonoff theorem. **[10]**

b) Show that arbitrary product of regular spaces is regular. **[6]**

Q8) a) State and prove Urysohn's lemma. **[10]**

b) State Tietze Extension Theorem. **[4]**

c) Define Quotient topology. **[2]**

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Total No. of Questions : 8] **[Total No. of Pages : 2**

P367

[4223] - 201
M.Sc.
MATHEMATICS
MT - 601 : Real Analysis - II
(Old Course) (Semester - II)

Time : 3 Hours] ***[Max. Marks :80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) If f is Riemann integrable on $[a, b]$, then prove that f is Lebesgue integrable. **[6]**

b) Let $f \in R_\alpha [a, b]$ and c be a real number. Prove that $cf \in R_\alpha[a,b]$ and $\int_a^b cf\, d\alpha = c \int_a^b f\, d\alpha.$ **[5]**

c) True or False? Justify.
If $f = g$ a.e. on $[a, b]$ and f is Riemann integrable on $[a, b]$, then g is also Riemann integrable on $[a, b]$. **[5]**

Q2) a) State and prove Lebesgue's Monotone Convergence Theorem. **[8]**

b) Give an example of a non-measurable set. **[8]**

Q3) a) If $f \in R_\alpha [a, b]$, then prove that $\alpha \in R_f [a, b]$ and
$\int_a^b f\, d\alpha + \int_a^b \alpha\, df = f(b)\alpha(b) - f(a)\alpha(a).$ **[6]**

b) Write the fourier series for the following function :
$f(x) = x$ for $x \in [-\pi, \pi]$ **[7]**

c) Define the outer measure. Show that the outer measure of $\{a\}$ is zero. **[3]**

Q4) a) If E and F are disjoint compact sets, then prove that
$m^*(E \cup F) = m^*(E) + m^*(F)$. **[6]**

b) Show that the Lebesgue integral $\int_0^\infty \frac{\sin x}{x}\, dx$ does not exist. **[6]**

c) Let $\{f_n\}$ be a sequence of measurable functions, then show that $\inf_n f_n$ is measurable. **[4]**

P.T.O.

Q5) a) If ϕ and ψ are integrable simple functions and α, β are real numbers, then prove that $\int (\alpha\phi + \beta\psi) = \alpha \int \phi + \beta \int \psi$. **[6]**

b) Suppose f is a non-negative and measurable function, then show that $\int f = 0$ if and only if $f = 0$ a.e. **[5]**

c) If F is a closed subset of a bounded open set G, then prove that $m^*(G/F) = m^*(G) - m^*(F)$. **[5]**

Q6) a) If f is a measurable function, then show that $|f|$ is measurable. Is the converse true? Justify. **[5]**

b) Let $\{E_n\}$ be a sequence of measurable sets. If $E_n \supset E_{n+1}$ for each n and $m(E_k)$ is finite for some k, then prove that $m\left(\bigcap_{n=1}^{\infty} E_n\right) = \lim_{n\to\infty} m(E_n)$ **[6]**

c) Show that $\lim_{n\to\infty} \int_0^1 f_n = 0$ where $f_n(x) = \dfrac{n\sqrt{x}}{1+n^2x^2}$. **[5]**

Q7) a) State and prove Fatou's Lemma. **[8]**

b) Let $1 < p < \infty$ and q be defined by $\dfrac{1}{p} + \dfrac{1}{q} = 1$.

If $f \in L_p(E)$ and $g \in L_q(E)$, then prove that $fg \in L_1(E)$ and $\left|\int_E fg\right| \le \int_E |fg| \le \|f\|_p \, \|g\|_q$. **[5]**

c) Give an example of absolutely continuous function. **[3]**

Q8) a) With usual notations, prove that $\|f_1 f_2\|_{BV} \le \|f_1\|_{BV} \|f_2\|_{BV}$. **[5]**

b) Let $f, g \in BV[a, b]$ and $a \le c \le b$, than prove that

$V_a^b(f+g) \le V_a^b(f) + V_a^b(g)$ and

$V_a^b(f) = V_a^c(f) + V_c^b(f)$. **[6]**

c) True or False? Justify.

A bounded continuous function is of bounded variation. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P368

[Total No. of Pages : 2

[4223] - 202

M.Sc.

MATHEMATICS

MT-602 : Differential Geometry

(2008 Pattern) (Semester -II)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let U be an open set in R^{n+1} and let $f : U \to R$ be a smooth function. Let $P \in U$ be a regular point of f and let $c = f(p)$. Prove that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{\perp}$ **[7]**

b) Let $g(x, y) = x^2 + 4xy + y^2$. Find minimum and maximum value of $g(x, y)$ on the unit circle. **[6]**

c) Define the term 'orientation' of an n-surface in R^{n+1}. **[3]**

Q2) a) Let S be an n-surface in R^{n+1}, oriented by the unit normal vector field N. Let $p \in S$ and $V \in S_p$. Prove that for every parametrized curve $\alpha : I \to S$ with $\dot{\alpha}(t_0) = V$ for some $t_0 \in I$. $\ddot{\alpha}(t_0) \cdot N(p) = L_p(V) \cdot V$. **[6]**

b) For the parametrized curve $\alpha(t) = \left(\dfrac{\sin t}{\sqrt{2}}, \dfrac{\sin t}{\sqrt{2}}, \cos t\right)$ on the unit 2 - sphere S^2, find the parallel transport of a vector in the tangent space at North Pole to the tangent space at the south pole. **[6]**

c) Describe the spherical image of the oriented n-surface $f^{-1}(1)$ where $f(x_1, x_2, ..., x_{n+1}) = x_2^2 + ... + x_{n+1}^2$, oriented by $\dfrac{\nabla f}{\|\nabla f\|}$. **[4]**

Q3) a) Define second fundamental form $\mathcal{S}_p$ of a surface at a point p and show that $\mathcal{S}_p$ is positive definite if and only if all the principal curvatures at P are positive. **[6]**

b) Find integral curve through (1, 0) of the vector field $X(p) = (p, X(p))$ where $X(x_1, x_2) = (-x_2, x_1)$. **[6]**

c) Show that the special linear group SL(2) is a 3 - surface in R^4. **[4]**

P.T.O.

Q4) a) Let C be a Connected oriented plane and let $\beta: I \to C$ be a unit speed global Parametrization of C. Prove that β is either one to one or periodic. **[6]**

b) Show that $\alpha(t) = (r\cos t, r\sin t, t)$ is a geodesic in the cylinder $x_1^2 + x_2^2 = r^2$. **[6]**

c) Find curvature of the circle $f^{-1}(r^2)$ where $f(x_1, x_2) = (x_1 - a)^2 + (x_2 - b)^2$ oriented by the outward normal $\frac{\nabla f}{\|\nabla f\|}$. **[4]**

Q5) a) Let S be an oriented n - surface in R^{n+1} which is convex at $P \in S$. Show that the second fundamental form $\mathcal{S}_p$ of S at P is semi - definite. **[6]**

b) Find Gaussian curvature of the cone $x_1^2 + x_2^2 - x_3^2 = 0,\ x_3 > 0$. **[6]**

c) For $f_1(x_1, x_2, x_3, x_4) = x_1^2 + x_2^2$ and $f_2(x_1, x_2, x_3, x_4) = x_3^2 + x_4^2$; prove that $S = f_1^{-1}(1) \cap f_2^{-1}(1)$ is a 2- surface in R^4. **[4]**

Q6) a) Let $S \subseteq R^{n+1}$ be a connected n-surface in R^{n+1}. Then prove that there exist on S, exactly two smooth unit normal vector fields N_1 and N_2; and $N_1(p) = -N_2(p)$ for all $p \in S$. **[6]**

b) Let S be an n-plane $a_1 x_1 + ... + a_{n+1} x_{n+1} = b$ in R^{n+1}, let $p, q \in S$ and let $v = (P, V) \in S_p$. Show that if α is any parametrized curve in S from p to q, then show that $P(v) = (q, v)$. **[6]**

c) If an n-surface S contains a line segment L, then show that L is a geodesic of S. **[4]**

Q7) a) Let S be an n-surface in R^{n+1} and let $f: S \to R^k$. Then S is smooth if and only if $f_0\ \phi : U \to R^k$ is smooth for each local parametrization $\phi : U \to S$. **[8]**

b) Let $\eta = -\frac{x_2}{x_1^2 + x_2^2} dx_1 + \frac{x_1}{x_1^2 + x_2^2} dx_2$ be a 1 – form on $R^2 - \{0\}$ and let C denote the ellipse $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} = 1$ oriented by its inward normal. Find $\int_c \eta$. Is η an exact form? Justify. **[8]**

Q8) a) Show that the weingarten map Lp is self - adjoint. **[8]**

b) Let S be a compact, connected, oriented n-surface in R^{n+1} whose Gauss - Kronecker curvature is nowhere zero. Prove that S is strictly convex. **[8]**

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Total No. of Questions : 5]

SEAT No. :

P372

[Total No. of Pages : 2

[4223] - 206

M.Sc.

MATHEMATICS

MT-606 : Object Oriented Programming with C++

(2008 Pattern) (Sem. - III)

Time : 2 Hours] ***[Max. Marks :50***

Instructions to the candidates:

1) ***Figures to the right indicate full marks.***
2) ***Attempt any two from questions no. 2, 3 and 4.***
3) ***Question 1 is compulsory and each subquestion carries 2 marks each.***
4) ***Question 5 is compulsory.***

Q1) Answer any 10 questions of the following : **[20]**

a) What is message passing?

b) What are C++ Key words?

c) Differentiate between CIN and COUT.

d) Name four operators in C++ not used in C.

e) Write down the syntax of delete operator.

f) Explain the term Reference variable.

g) Write down output of following program.

```
int i = 15
Cout << dec << i << end1;
Cout << Hex << i << end1;
Cout << Oct << i << end1;
```

h) Write down output of following program

```
Void func (int a = 1, int b = 2, int C = 3)
{
   Cout <<"  a =   "<< a <<" b = "<< b <<" c = "<< c << end1;
}
   int main ()
   {
      func ();
      func (10);
      return O;
   }
```

P.T.O.

i) Write down advantages of Macros.

j) What are disadvantages of inline function?

k) How to declare constructor?

l) What is a friend function?

Q2) a) What are benifits of object oriented programming? **[5]**

b) Write a note on type conversion. **[5]**

Q3) a) What is difference between struct in C and struct in C++. **[5]**

b) Write a note on nesting of classes. **[5]**

Q4) a) What are methods of passing information in C++. **[5]**

b) Write a note on function overloading. **[5]**

Q5) a) Write a program using constructor. **[5]**

b) Write a note on overloading. Insertion Operator (< <). **[5]**

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Total No. of Questions : 8]

SEAT No. :

P364

[Total No. of Pages : 4

[4223] - 103

M.Sc.

MATHEMATICS

MT- 503 : Linear Algebra

(2008 Pattern) (Semester - I)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***
3) ***All questions carry equal marks.***
4) ***Neat diagarams must be drawn wherever necessary.***

Q1) a) Prove that a linearly independent subset of a finite dimensional vector space can be extended to form a basis of the vector space. **[6]**

b) If V is a subspace of R^5 generated by $v_1 = [1\ 3 - 2\ 2\ 3]^t, v_2 = [2\ 7\ - 5\ 6\ 5]^t$

$v_3 = [3\ 6 - 3\ 0\ 13]^t$ and if W is a subspace generated by $w_1 = [1\ 3\ 0\ 2\ 1]^t, w_2 = [5\ 16 - 3\ 12\ 6]^t$, $w_3 = [3\ 8\ 3\ 4\ 2]^t$ then find a basis and dimension of the subspace $V \cap W$. **[6]**

c) Prove or disprove

i) R, the set of all real Numbers, is finite dimensional vector space over Q, set of all rational numbers.

ii) There exist a vector space with 9 elements. **[4]**

Q2) a) If V and V' are two vector spaces over the same field K and B is a finite basis of V. If f: B $\rightarrow$V' is an arbitrary mapping then prove that there exists a unique T$\in$ L (V,V') such that T|B = f. **[5]**

b) Let V and V' be two vector spaces over the same field K with $B = \{v_1, v_2 ......, ...v_n\}$ as a basis of V. Also let T$\in$ L (V,V'). Prove that if T is injective then dim $V \leq \dim V'$. What happens if T is surjective? **[5]**

P.T.O.

c) Prove or disprove. **[6]**

i) If V is a vector space of dimension n over the field K and W is a vector space of same dimension over the field K' (K $\neq$ K') then $V \simeq V'$.

ii) If T is a linear operator on a vector space V and if $m \in N$ is the nilpotency index of T then m cannot exceed the dimension of V.

Q3) a) If W is any subspace of a finite dimensional vector space V over the field K then prove that

dim V = dim W + dim W^0 where W^0 is the annihilator of W. **[6]**

b) Show that $B = \left\{ [2\ 1\ 1]^t, [3\ 4\ 1]^t, [2\ 2\ 1]^t \right\}$ is a basis of R^3 over R. Find a basis of $(R^3)^*$ dual to B. **[6]**

c) Find non-zero-subspaces W_1, W_2 and W_3 of R^3 such that $R^3 = W_1 + W_2 + W_3$, $W_i \cap W_j = \{0\}$ for $i \neq j$ but $R^3 \neq W_1 \oplus W_2 \oplus W_3$. **[4]**

Q4) a) If X and Y are subspaces of a vector space V such that $\frac{V}{X}$ and $\frac{V}{Y}$ are finite dimensional then prove that the quotient space $\frac{V}{X \cap Y}$ is also finite dimensional. **[6]**

b) Let $V = K_3[x]$ be the space of all polynomials of degree at most 3 and let $T: V \to V$ be the linear transformation map given by $T(f(x)) = f'(x)$. Find the minimal polynomial of T. **[5]**

c) If $A = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{pmatrix} \in R^{3\times3}$ then show that A is not diagonalizable. What happens if $A \in C^{3\times3}$? **[5]**

Q5) a) Define generalized eigen vector of a linear transformation T. Prove that a Jordan chain consists of linearly independent vectors. **[6]**

b) Consider the following matrix $A = \begin{pmatrix} -2 & 5 & 1 & 0 \\ -2 & 4 & 1 & 0 \\ -1 & 2 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{pmatrix}$

Find algebraic multiplicity and geometric multiplicity of eigen values of A and hence find Jordan form A. **[10]**

Q6) a) Let v be an inner product space over F where F is either R or $\mathbb{C}$ and $u,v \in V$ prove that **[6]**

i) $|(u,v)| \leq \|u\| \, \|v\|$

ii) $\|u+v\| \leq \|u\| + \|v\|$

iii) $\big| \|u\| - \|v\| \big| \leq \|u-v\|$.

b) Let V =R_2 (t) be the space of all polynomials of degree at most 2 over R. Define an inner product by $(f(t), g(t)) = \int_0^1 f(t)\, g(t)\, dt$. Obtain an orthonormal basis from the basis $\{1, t, t^2\}$ of V. **[6]**

c) i) If u and v are vectors in an inner product space such that $\|u+v\| = 8$ $\|u-v\| = 6$ and $\|u\| = 7$ *find* $\|v\|$.

ii) If $W = \left\{ f : R \to R \,|\, f^{''}(x) + 4f = 0 \right\}$ is a vector space over R. Find the dimension of W. **[4]**

Q7) a) State and prove Riesz representation theorem. **[8]**

b) If W_1 and W_2 are subspaces of a vector space V then prove that $(W_1 + W_2)^{\perp} = W_1^{\perp} \cap W_2^{\perp}$ **[6]**

c) If $u, v \in V$ are orthonormal then prove that $\|u\| \leq \|u - \mu v\|$ for some $\mu \in K$ where V is an product space over the field K. **[2]**

Q8) a) Define the adjoint of transformation T. If V and W are two finite dimensional inner product spaces over the field F. Then prove the following.

i) If $S, T \in L(V, W)$ then $(S + T)^* = S^* + T^*$

ii) If $T \in L(V)$ and if T is invertible then $(T^*)^{-1} = (T^{-1})^*$ **[6]**

b) If T is a normal operator on an inner product space V then prove the following.

i) $\| Tv \| = \| T^* v \|$ for all $v \in V$

ii) If for $v \in V, Tv = \lambda v, \lambda \in F$ then $T^* v = \bar{\lambda} v$

iii) Eigenvectors corresponding to distinct eigenvalues of T are orthogonal. **[6]**

c) If the matrix A is unitary then show that A^t, A^{-1} are also unitary. **[4]**

■■■

Total No. of Questions : 8]

SEAT No. :

P365

[Total No. of Pages : 2

[4223] - 104
M.Sc.
MATHEMATICS
MT- 504 : Number Theory
(2008 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) State and prove the law of quadratic reciprocity. **[6]**

b) Let $f(x) = x^2 + x + 7$. Find all roots of $f(x) \equiv 0$ (mod 189), given that the roots (mod 27) are 4, 13 and 22. **[5]**

c) What are the last two digits of 3^{541}. **[5]**

Q2) a) If $n = 2^{\alpha} \Pi p^{\beta} \; \Pi q^{\gamma}$ (Where p, q are primes and p being $4k + 1$ type and q being $4k + 3$ type) is a canonical factorization, then prove that n is sum of two squares iff each γ is even. **[6]**

b) What is the remainder when the following sum is divided by 4

$1^5 + 3^5 + ... + 97^5 + 99^5$ **[5]**

c) Explain Pollard ρ method to locate proper divisors of an integer. **[5]**

Q3) a) Every Euclidean quadratic field has the unique factorization property. **[8]**

b) Let $Q(\sqrt{m})$ have the unique factorization property. Then prove that any prime π in $Q(\sqrt{m})$ there corresponds one and only one rational prime p such that $\pi | p$ **[6]**

c) Prove that $1 + i$ is a prime in Z (i). **[2]**

P.T.O.

Q4) a) If ϕ is Euler totient function on set of positive integers and m, n are integers greater than 1 such that $\phi(nm) = \phi(m)$ then prove that m is odd and $n = 2$. **[6]**

b) State and prove the Hensel's lemma. **[5]**

c) Define Mobius mu function $\mu(n)$ for an integer n and prove that if $F(n) = \sum_{d|n} f(d)$ for every positive integer n, then $f(n) = \sum_{d|n} \mu(d) F(n/d)$. **[5]**

Q5) a) If g and h are two distinct primitive roots modulo a prime p then prove that gh is not a primitive root mod p. **[6]**

b) If p is a prime then prove that there exist $\phi(p-1)$ primitive roots $(mod\ p)$. **[5]**

c) Prove that $(p-1)! + 1$ is power of a prime iff $p = 2,3$ or 5. **[5]**

Q6) a) State and prove the Gauss lemma. **[6]**

b) Find whether $x^2 \equiv 219(mod\ 419)$ has a solution? **[5]**

c) Find all the odd primes $x^2 \equiv 5(mod\ p)$. **[5]**

Q7) a) Prove that for each positive integer n, $d(n) = \prod_{p^{\alpha}||n} (\alpha + 1)$, where $d(n)$ is the number of positive divisors of n. **[6]**

b) Prove that the product of primitive polynomials is primitive. **[5]**

c) Prove that the minimal equation of an algebraic integer is monic with integer coefficients. **[5]**

Q8) a) Prove that the reciprocal of a unit is unit and also prove that the units of an algebraic number field form a multiplicative group. **[6]**

b) Prove that if a belongs to exponent 3 modulo a prime p then $1 + a + a^2 \equiv 0(mod\ p)$ and $1 + a$ belongs to exponent 6. **[5]**

c) Find all odd primes p for which 2 is square $(mod\ p)$. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P366

[Total No. of Pages : 3

[4223] - 105
M.Sc.
MATHEMATICS
MT- 505 : Ordinary Differential Equations
(2008 Pattern) (Semester - I)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Find the general solution of $y'' - y' - 2y = 4x^2$. **[5]**

b) If $y_1(x)$ and $y_2(x)$ are any two solutions of equation $y'' + P(x)y' + Q(x)y = 0$ on $[a, b]$ then prove that their Wornskian W=(y_1, y_2) is identically equal zero or nerver zero on [a,b]. **[5]**

c) Verify that $y_1 = x^2$ is one solution of $x^2 y'' + xy' - 4y = 0$ and find y_2 and general solution. **[6]**

Q2) a) State and prove Sturm Comparison theorem. **[8]**

b) Find the general solution of : $x(1-x)y'' + \left(\frac{3}{2} - 2x\right)y' + 2y = 0$ **[8]**

Q3) a) Verify that origin is regular, singular point and calculate two independent Frobenius Series solution for the equation : $2xy'' + (x+1)y' + 3y = 0$ **[8]**

b) Let u(×) be any non-trivial solution of $u'' + q(x)u = 0$ where q(×) > 0 for all × >0.

If $\int_1^\infty q(x)dx = \infty$, then prove that u(×) has infinitely many zeroes on the positive X-axis. **[8]**

P.T.O.

Q4) a) Find the general solution of the system:

$$\frac{dx}{dt} = 3x - 4y ; \frac{dy}{dt} = x - y$$ **[8]**

b) Locate and classify the singular points on the x-axis of $x^2(x^2-1)^2 y'' - x(1-x)y' + 2y = 0$. **[4]**

c) find the general solution of $y'' - 3y' + 2y = 14\sin(2x) - 18\cos(2x)$. **[4]**

Q5) a) If $a_1b_2 - a_2b_1 \neq 0$, then show that the system

$\frac{dx}{dt} = a_1x + b_1y ; \frac{dy}{dt} = a_2x + b_2y.$ has infinitely many critical points, none of which are isolated. **[6]**

b) Show that $y(x) = c_1 \sin(x) + c_2 \cos(x)$ is the general solution of $y'' + y = 0$, on any interval and find the particular solution for which $y(0) = 2$ and $y'(0) = 3$. **[4]**

c) Find the nature and stability properties of critical point (0,0) for:

$$\frac{dx}{dt} = -3x + 4y ; \frac{dy}{dt} = -2x + 3y$$ **[6]**

Q6) a) Find the general solution near $x = 0$ of the hyper-geometric equation: $x(x-1)y'' + [c - (a+b+1)x]\, y' - aby = 0$, Where a, b and c, are constants. **[8]**

b) Find the exact solution of initial value problem: $y' = y^2, y(0) = 1$; starting with $y_0(x) = 1$. Apply Picard's method to calculate $y_1(x), y_2(x), y_3(x)$ and compare it with the exact solution. **[8]**

Q7) a) Show that the function $f(x,y) = xy^2$ satisfies Lipschitz condition on any rectangle $a \leq x \leq b$ *and* $c \leq y \leq d$; but it does not satisfy a Lipschitz condition on any strip $a \leq x \leq b$ *and* $-\infty < y < \infty$. **[10]**

b) Solve the following initial value problem: **[6]**

$$\frac{dy}{dx} = Z \quad y(0) = 1 ;$$

$$\frac{dz}{dx} = -y \quad z(0) = 0$$

Q8) a) Find the general solution of $(1-x^2)y''-2xy'+p(p+1)y=0$, about $x=0$ by power series method. **[8]**

b) If m_1 and m_2 are roots of the auxiliary equation of the system:

$$\frac{dx}{dt}=a_1x+b_1y;\frac{dy}{dt}=a_2x+b_2y.$$ **[8]**

Which are real, distinct and of same sign, then prove that the critical point (0,0) is a node.

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Total No. of Questions : 8]

SEAT No. :

P369

[Total No. of Pages : 2

[4223] - 203
M.Sc.
MATHEMATICS
MT- 603 : Groups and Rings
(2008 Pattern) (Sem. - II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Solve any five questions.***
2) ***Marks at right indicate full marks.***

Q1) a) Find centre of S_3. **[3]**

b) Show that intersection of subgroups is a subgroup. What about union? **[5]**

c) State and prove fundamental theorem of cyclic groups. **[8]**

Q2) a) Determine the subgroup lattice for Z_{12}. **[4]**

b) Show that group of order prime is cyclic. What about group of order p^2, p prime? **[8]**

c) Express (1 2 3 4 5) in S_7 as product of 2 - cycles. **[4]**

Q3) a) Prove that alternating group with n elements has order $\frac{n!}{2}$. **[5]**

b) Find the order of permutation (1 2 4) (3 5 7 8). **[3]**

c) State and prove Cayley's theorem. **[8]**

Q4) a) Find the action of the inner automorphism of D_4 induced by R_{90}. **[5]**

b) Show that for every positive integer n, Aut (Z_n) is isomorphic to U(n). **[8]**

c) Find cosets of $H = \{(1), (12)\}$ in S_3. **[3]**

P.T.O.

Q5) a) Find orbit and stabilizer of 7 in G = {(1), (132) (465) (78), (132) (465), (123) (456), (123) (456) (78), (78)}. **[4]**

b) Prove that group of rotations of a cube is isomorphic to S_4. **[6]**

c) Show that G is isomorphic to subgroup of $G \oplus H$. **[6]**

Q6) a) State and prove G/Z theorem. **[4]**

b) Show that Kernel of homomorphism is normal subgroup of domain group. **[6]**

c) Show that the only group of order 255 is Z_{255}. **[6]**

Q7) a) State Sylow's theorem for Abelian group. **[6]**

b) State and prove fundamental theorem of finite Abelian Groups. **[10]**

Q8) a) Show that G is isomorphic to subgroup of $G/H \oplus G/K$ where H and K are normal subgroups of G with $H \cap K = \{e\}$. **[8]**

b) State and prove first isomorphism theorem for groups. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P370

[Total No. of Pages : 3

[4223] - 204
M.Sc.
MATHEMATICS
MT- 604 : Complex Analysis
(2008 Pattern) (Semester - II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *All questions carry equal marks.*

Q1) a) Let $f(z)=\sum_{n=0}^{\infty} a_n z^n$ have radius of convergence R>0. Prove that for each k $\geq$ 1 the series $\sum_{n=k}^{\infty} n(n-1)\ ----\ (n-k+1)a_n z^{n-k}$ has radius of convergence R. **[6]**

b) Under stereographic projection which subsets of S = $\{(x_1, x_2, x_3) \in R^3 \mid x_1^2 + x_2^2 + x_3^2 = 1\}$ corresponds to the real and imaginary axes in C **[5]**

c) (i) Define Analytic function. If f: G $\rightarrow$ C is differentiable at 'a' in G then prove that *f* is continuous at 'a'.

(ii) Show that (sinz)' = cos z **[5]**

Q2) a) If G is open and connected and f : G $\rightarrow$C is differentiable with $f'(z) = 0 \ \forall z \in G$ then prove that *f* is constant. **[6]**

b) Let G be either the whole plane C or some open disk. If u: G $\rightarrow$ R is a harmonic function then prove that u has a harmonic conjugate. **[5]**

c) Let $\gamma(t) = 1 + e^{it}$ for $0 \leq t \leq 2\pi$. Find $\int_{\gamma}\left(\frac{z}{z-1}\right)^n dz$ for all positive integer n. **[5]**

P.T.O.

Q3) a) Let z_1, z_2, z_3, z_4 be four distinct points in $\mathbb{C}_\infty$ then prove that (z_1, z_2, z_3, z_4) is a real number iff all four points lie on a circle. **[6]**

b) Find an analytic function $f: G \to C$ where

$G = \{Z : \operatorname{Re} z > 0\}$ s.t $f(G) = D = \{z : |z| < 1\}$ **[6]**

c) Find the radius of convergence of the power series $\sum_{k=0}^{\infty} \frac{k}{6^k} Z^k$ **[4]**

Q4) a) Let f: G $\to$C be analytic and suppose $\overline{B(a; r)} \subset G$ $(r>0)$. If $\gamma(t) = a + re^{it}$, $0 \le t \le 2\pi$ then prove $f(z) = \frac{1}{2\pi i} \int \frac{f(w)}{w - z}$ dw for $|z - a| < \gamma$ **[8]**

b) Prove $\int_0^{2\pi} \frac{e^{is}}{e^{is} - z}$ ds $= 2\pi$ $|z| < 1$ **[5]**

c) Let f be analytic in B (a ; R) and suppose **[3]**

$|f(z)| \le M \;\forall\; Z \in B(a; R)$. Prove that $|f^{(n)}(a)| \le \frac{n : M}{R^n}$

Q5) a) State and prove Fundamental theorem of Algebra. **[6]**

b) If $\gamma : [0,1] \to C$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then prove that $\frac{1}{2\pi i} \int_\gamma \frac{dz}{z - a}$ is an integer. **[5]**

c) Evaluate the integral $\int_\gamma \frac{\sin z}{z^2 + 1}$ dz where $\gamma(t) = 2e^{it}$ $0 \le t \le 2\pi$ **[5]**

Q6) a) Let γ be a rectifiable curve and suppose ϕ is a function defined and continuous on $\{\gamma\}$. For each $m \ge 1$ let

$F_m(z) = \int \phi(w)\,(w - z)^{-m}$ dw for $Z \notin \{\gamma\}$ then prove that each F_m is analytic on C-$\{\gamma\}$ and $F'_m(z) = m\, F_{m+1}(z)$. **[6]**

b) Let G be a region and let f be an analytic function on G with zeros $a_i, \ldots, a_m$ repeated according to multiplicity. If γ is a closed rectifiable curve in G which does not pass through any point a_k and if $\gamma \approx 0$ then prove that

$$\frac{1}{2\pi i} \int_{\gamma} \frac{f'(z)}{f(z)}\, dz = \sum_{k=1}^{M} \eta(\gamma; a_k)$$ **[5]**

c) Let f be analytic in B(a;R) and suppose that f(a) = 0. Show that 'a' is a zero of multiplicity m iff $f^{(m-1)}(a) = \text{------} = f(a) = c$ and $f^{(m)}(a) \neq 0$ **[5]**

Q7) a) If f has an isolated singularity at 'a' then prove that the point z =a is a removable singularity iff $\lim\limits_{z \to a} (z-a) f(z) = 0$ **[8]**

b) State and prove Casoroti-Weierstrass theorem. **[6]**

c) State Cauchy Residue theorem. **[2]**

Q8) a) State and prove Rouche's theorem. **[6]**

b) Let G be a region in C and f an analytic function on G. Suppose there is a constant M such that limsup $|f(z)| \leq M \;\forall\; a \in \partial_{\infty} G$

Prove that $|f(z)| \leq M \;\forall\; Z \in G$ **[6]**

c) Find all possible values of $\int_{\gamma} \frac{dz}{1+Z^2}$ where γ is any closed rectifiable curve in C not passing through $\pm i$. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P371

[Total No. of Pages : 3

[4223] - 205
M.Sc.
MATHEMATICS
MT- 605 : Partial Differential Equations
(2008 Pattern) (Sem. - II)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Find the general solution of $x^2 p + y^2 q = (x+y)z$. **[5]**

b) Prove that the Pfaffian differential equation

$\overline{X}.d\overline{r} = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0$ is integrable if and only if $\overline{X}.Curl\,\overline{X} = 0$. **[8]**

c) Eliminate the arbitrary function F from the equation $z = xy + F(x^2 + y^2)$ and find the corresponding partial differential equation. **[3]**

Q2) a) Explain the method of solving following first order partial differential equation.

(i) $f(p,q) = 0$,

(ii) $z = px + qy + g(p,q)$. **[6]**

b) Find a complete integral of $p^2 + q^2 = x + y$. **[5]**

c) Solve the differential equation $z^2(p^2 z^2 + q^2) = 1$. **[5]**

Q3) a) Prove that the differential equation $dz = \phi(x,y,z)dx + \psi(x,y,z)dy$ is integrable if and only if $\frac{\partial(f,g)}{\partial(x,p)} + p\frac{\partial(f,g)}{\partial(z,p)} + \frac{\partial(f,g)}{\partial(y,q)} + q\frac{\partial(f,g)}{\partial(z,q)} = 0$. **[6]**

P.T.O.

b) Find a complete integral of the equation $p^2x + q^2y = z$ by Jacobi's method. **[5]**

c) Find the integral surface of the equation $(2xy - 1)p + (z - 2x^2)\, q = 2(x - yz)$, Which passes through the line $x_0(s) = 1, y_0(s) = 0$ and $z_0(s) = s$. **[5]**

Q4) a) Find the characteristic strips of the equation $z + px + qy = 1 + pq\, x^2 y^2$, passing through the initial data curve $C : x_0 = S, y_0 = 1, z_0 = -S$. **[6]**

b) Reduce the equation $\dfrac{\partial^2 u}{\partial x^2} - x^2 \dfrac{\partial^2 u}{\partial y^2} = 0$ to a canonical form. **[5]**

c) Obtain the d' Alembert's solution of the following one-dimensional wave equation: $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}, -\infty < x < \infty, t > 0,$

$y(x,0) = f(x), y_t(x,0) = g(x), \; -\infty < x < \infty.$ **[5]**

Q5) a) Using the method of separation of variables solve the following wave equation: **[10]**

$y_{tt} - c^2 y_{xx} = 0, \qquad 0 < x < l, t > 0$

$y(x,0) = f(x), \qquad 0 \le x \le l,$

$y_t(x,0) = g(x), \qquad 0 \le x \le l,$

$y(0,t) = y(l,t) = 0, \qquad t > 0.$

b) Suppose that $u(x,y)$ is a harmonic function in a bounded domain D and is continuous on $\bar{D} = B \cup D$. Then prove that u attains its minimum on the boundary B of D. **[3]**

c) Classify the following equation into hyperbolic, parabolic or elliptic type: $u_{xx} + 2u_{yz} + \cos x\, u_z - e^{y^2} u = \cosh z.$ **[3]**

Q6) a) Using Duhamel's principle find the solution of non - homogeneous heat equation $u_t + ku_{xx} = F(x,t), -\infty < x < \infty, t > 0,$

$u(x,0) = 0, \; -\infty < x < \infty.$ **[8]**

b) Solve $u_{xx} + u_{yy} = 0, 0 < x < a, 0 < y < b$

with the boundary conditions

$u(x,0) = f(x), \quad 0 \le x \le a,$

$u(x,b) = 0, \quad 0 \le x \le a,$

$u(0,y) = 0, \quad 0 \le y \le b,$

$u(a,y) = 0, \quad 0 \le y \le b.$ **[8]**

Q7) a) Prove that the solution $u(x,t)$ of the differential equation

$u_t - ku_{xx} = F(x,t), 0 < x < l, t > 0,$

with the initial condition

$u(x, 0) = f(x), 0 \le x \le l,$

and the boundary conditions

$u(0,t) = u(l,t) = 0, t \ge 0$ is unique. **[8]**

b) By the method of characteristics, find the integral surface of $pq = xy$ which passes through the curve $z = x, y = 0.$ **[6]**

c) State Harnack's theorem. **[2]**

Q8) a) Obtain the singular solution of

$z - px - qy - p^2 - q^2 = 0.$ **[5]**

b) Prove that the solution for the Dirichlet problem for a circle of radius a is given by the poisson integral formula

$$u(\rho,\theta) = \frac{1}{2\pi}\int_0^{2\pi}\frac{1-\rho^2}{1-2\rho\cos(\theta-\tau)+\rho^2}f(\tau)d\tau$$ **[8]**

c) Classify the following equations into linear, semi-linear and quasi-linear: **[3]**

(i) $yp - xq = xyz + x,$

(ii) $e^x p - yxq = xz^2,$

(iii) $(x^2 + z^2)p - xyq = z^3x + y^2.$

Total No. of Questions : 8]

SEAT No. :

P373

[Total No. of Pages : 2

[4223] - 301
M.Sc.
MATHEMATICS
MT- 701 : Functional Analysis
(2008 Pattern) (Semester - III)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Give example of a space with two non-equivalent norms on it. Justify. **[6]**

b) State and prove the Uniform boundedness principle. **[8]**

c) Show that the norm of an isometry is 1. **[2]**

Q2) a) Let M be a closed linear subspace of a normed linear space N. If a norm of a coset $x + M$ in the quotient space N/M is defined by $|||x + M||| = \inf\{\| x + m\| : m \in M\}$, then prove that N/M is a normed linear space. Further if N is Banach, then prove that N/M is also a Banach space. **[8]**

b) Show that an operator T on a finite dimensional Hilbert space H is normal if and only if its adjoint T* is a polynomial in T. **[6]**

c) A linear operator S : $l^2 \to l^2$ is defined by

S $(x_1, x_2, ...) = (0, x_1, x_2, ...)$. Find its adjoint S^*. **[2]**

Q3) a) Prove that an infinite dimensional Banach space cannot have a denumerable Hamel basis. **[7]**

b) Show that every positive operator on a finite dimensional Hilbert space has a unique positive square root. **[5]**

c) Let $\{A_n\}$ be a sequence in B (H) and A $\in$ B (H) such that $\|A_n - A\| \to 0$ as $n \to \infty$. If each A_n is self-adjoint, then show that A is self-adjoint. **[4]**

Q4) a) Show that $||T^*|| = ||T||$ and $||T^*T|| = ||T||^2$. **[6]**

b) i) Let X and Y be normed spaces. If X is finite dimensional, then show that every linear transformation from X to Y is continuous. **[4]**

ii) Give an example of a discontinuous linear transformation. **[4]**

c) Let H be a 2-dimensional Hilbert space. Let the operator T on H be defined by $Te_1 = e_2$ and $Te_2 = -e_1$. Find the spectrum of T. **[2]**

Q5) a) If T is an operator on a Hilbert space H, then prove that T is normal if and only if its real and imaginary parts commute. **[6]**

b) Let M be a closed linear subspace of a normed linear space N and T be the natural mapping of N onto N/M defined by $\mathrm{T}(x) = x + M$. Show that T is a continuous linear transformation for which $||T|| \leq 1$. **[6]**

c) Find $M^{\perp}$ if $M = \{(x, y) : x + y = 0\} \subset \mathbf{R}^2$. **[4]**

Q6) a) Show that the unitary operators on a Hilbert space H form a group. **[4]**

b) If T is an operator on a Hilbert space H for which $\langle Tx, x\rangle = 0$ for all $x \in H$, then prove that $T = 0$ **[6]**

c) Let X be a normed space over C. Let $0 \neq a \in X$. Show that there is some functional f on X such that $f(a) = ||a||$ and $||f|| = 1$. **[6]**

Q7) a) Let H be a Hilbert space and f be a functional on H. Prove that there exists a unique vector y in H such that $f(x) = \langle x,y\rangle$ for every $x \in H$. **[8]**

b) Let $X = C^1([0,1])$ and $Y = C([0,1])$, both with sup norm. Let $F : X \to Y$ be defined as $F(\mathrm{g}) = g'$. Show that F is linear, closed but not continuous. Explain why the closed graph theorem is not applicable. **[8]**

Q8) a) State and prove the Open Mapping Theorem. **[8]**

b) Let T be a normal operator on H with spectrum $\{\lambda_1, \lambda_2, \ldots \lambda_m\}$. Show that T is self-adjoint if and only if each λ_i is real. **[4]**

c) Let T be an operator on H. If T is non-singular, then show that $\lambda \in \sigma(T)$ if and only if $\lambda^{-1} \in \sigma(T^{-1})$. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P380

[Total No. of Pages : 2

[4223] - 402
M.Sc.
MATHEMATICS
MT - 802 : Combinatorics
(2008 Pattern) (Sem. - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) How many different numbers can be formed by the product of two or more of the numbers 3, 4, 4, 5, 5, 6, 7, 7, 7 ? **[6]**

b) Find ordinary generating function whose coefficient a_r equals $(r + 1)\, r\, (r - 1)$ and hence evaluate the sum $3 \times 2 \times 1 + 4 \times 3 \times 2 + \ldots.. + (n+1)\, n\, (n - 1)$. **[6]**

c) There are 15 different apples and 10 different pears. How many ways are there for Jack to pick an apple or a pear and then for Jill to pick an apple and a pear? **[4]**

Q2) a) How many ways can a committee be formed from four men and six women with **[6]**

i) At least two men and at least twice as many women as men.

ii) Five people, and not all of the three O' Hara sisters can be on the committee?

b) How many arrangements of MISSISSIPPI are there with no pair of consecutive S's? **[6]**

c) Find a rook polynomial for a full n×n board. **[4]**

Q3) a) Use generating functions to find the number of ways to select 10 balls from a large pile of red, white and blue balls if the selection has even number of blue balls. **[6]**

b) How many arrangements of six 0's, five 1's and four 2's are there in which **[6]**

i) The first 0 precedes the first 1?

ii) The first 0 precedes the first 1, which precedes the first 2 ?

c) Solve the recurrence relation $a_n = 2a_{n-1} + 2n^2,\ a_0 = 3$ **[4]**

P.T.O.

Q4) a) Prove by combinatorial argument that $C(n, 1) + 6\,C(n, 2) + 6\,C(n,3) = n^3$ and evaluate $1^3 + 2^3 + 3^3 + ... + n^3$. **[6]**

b) How many 10-letter words are there in which each of the letters e, n, r, s occur **[6]**

i) at least once ? ii) at most once ?

c) How many 8 - digit sequences are there involving exactly six different digits ? **[4]**

Q5) a) Find a recurrence relation for a_n, the number of n - digit ternary sequences without any occurrence of the sub sequence "012". **[6]**

b) How many ways are there to split 6 copies of one book, 7 copies of a second book, and 11 copies of a third book between two teachers if each teacher gets 12 books and each teacher gets at least 2 copies of each book ? **[6]**

c) How many r - digit ternary sequences are there with at least one 0 and at least one 1 ? **[4]**

Q6) a) Use generating functions to solve the set of simultaneous recurrence relations. $a_n = a_{n-1} + b_{n-1} + c_{n-1}$, $b_n = 3^{n-1} - c_{n-1}$, $c_n = 3^{n-1} - b_n - 1$
$a_1 = b_1 = c_1 = 1$ **[10]**

b) Use inclusion exclusion formula to find how many different integer solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 20$, $0 \leq x_i \leq 8$ **[6]**

Q7) a) How many arrangements are there of MURMUR with no pair of consecutive letters the same? **[6]**

b) Using generating functions, solve the recurrence relation $a_n = a_{n-1} + n(n-1)$, $a_0 = 1$ **[6]**

c) Show that given any set of seven distinct integers, there must exist two integers in this set whose sum or difference is a multiple of 10. **[4]**

Q8) a) Five officials O_1, O_2, O_3, O_4, O_5 are to be assigned to five different city cars: an Escort, a lexus, a Nissan, a Taurus and a volvo.
O_1 will not drive Lexus and Nissan; O_2 will not drive escort or Taurus ; O_3 will not drive Escort and Lexus; O_4 will not drive car Nissan and O_5 will not drive volvo. How many ways are there to assign 5 cars to these 5 officials ? **[8]**

b) Solve the recurrence relations when $a_0 = 1$ **[8]**

i) $a_n^2 = 2a_{n-1}^2 + 1$

ii) $a_n = -n\, a_{n-1} + n!$

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Total No. of Questions : 8]

P380 [Total No. of Pages : 2

[4223] - 402
M.Sc.
MATHEMATICS
MT - 802 : Hydrodynamics
(Old Course) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates:

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Derive equation of continuity in Eulerian approach. **[8]**

b) A two dimensional incompressible flow field has the x component of velocity given by $u = e^{-x}(x \sin y - y \cos y)$. Determine the y component of velocity v. Is this flow irrotational ? Justify. **[8]**

Q2) a) A 3 D velocity field is given by $u = xy^2 t, v = \frac{1}{3} y^3 t^3, w = \frac{1}{2.} xyz^2 t^2$. Determine the convective, local and total acceleration at (1, 1, 2) at t = 1 s. **[8]**

b) Define stream lines and path lines. Given the velocity $\overline{q} = (1+t)x\hat{i} + (2+t)y\hat{j}$, find the equation of path line and stream line passing through (1, 2) at t=0. **[8]**

Q3) a) If the fluid motion is irrotational, show that velocity is derivable from potential function. **[7]**

b) The velocity components of certain flow are $u = -\frac{ax+by}{x^2+y^2}, v = \frac{bx+ay}{x^2+y^2}$ where a and b are constants. Find vorticity and calculate the circulation about the circle $x^2 + y^2 = c^2$. **[9]**

Q4) a) Define Stokes stream function $\psi(r, \theta)$ in spherical polar co-ordinates for the axisymmetric flow of an incompressible fluid. Determine the stream function corresponding to a uniform stream U parallel to the axis $\theta = 0$. **[10]**

P.T.O.

b) Discuss the flow for which $w = Uz^n$ where U is constant and $n>0$, a real number. **[6]**

Q5) a) Derive Bernoulli's equations for unsteady flow. **[8]**

b) In the cylindrical system (r, θ, z) the radial component of velocity of a two dimensional irrotational flow is given by $u(r, \theta) = \frac{3}{2}Ur^{3/2}\cos\frac{3\theta}{2}$.

Determine the transverse component of velocity. **[8]**

Q6) a) The velocity components of a flow expressed in spherical coordinates (r, θ, ϕ) are given by

$u = U\left(1 - \frac{3a}{r} + \frac{a^3}{2r^3}\right)\cos\theta, v = U\left(-1 + \frac{3a}{4r} + \frac{a^3}{4r^3}\right)\sin\theta$ where U and a are constants. Find the strain rate tensor. **[8]**

b) State and prove Blasius theorem. **[8]**

Q7) a) Determine the complex potential of a simple source. **[4]**

b) Show that the complex potential of the motion due to a uniform stream and any number of sources are additive provided no boundaries are present in the liquid. **[6]**

c) Show that the image of a doublet of strength μ at a distance $f > a$ from the centre of circle of radius a is a doublet at the inverse point of strength $\frac{\mu a^2}{f^2}$ and the axis of the doublet and its image are antiparallel. **[6]**

Q8) Write explanatory notes on <u>any two</u> : **[16]**

a) Kelvins minimum energy theorem

b) Lagrangian and Eulerian method

c) Kutta Joukowski theorem

d) Karman vortex street

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Total No. of Questions : 8]

P381

SEAT No. :

[Total No. of Pages : 2

[4223] - 403
M.Sc.
MATHEMATICS
MT-803 : Differential Manifolds
(2008 Pattern) (Semester-IV)

Time : 3 Hours] *[Max. Marks :80*

Instructions to the candidates:

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Let M be a compact k-manifold in R^n and $h : R^n \to R^n$ be an isometry. If $N = h(M)$, then show that M and N have the same volume. **[8]**

b) Give an example of a 1-manifold in R^2 which cannot be covered by a single coordinate patch. **[4]**

c) Let f and g be tensors on R^4 given by $f(X, Y, Z) = 3x_2y_1z_1 - x_1y_3z_2$, $g = \phi_{3,1} - 5\phi_{2,1}$. Express $f \otimes g$ as a linear combination of elementary 5-tensors. **[4]**

Q2) a) If V is a vector space of dimension n, find the dimension of $A^k(V)$, the space of alternating k - tensors on V. Justify. **[7]**

b) Define orientation of a manifold M and induced orientation on ∂ M. **[5]**

c) Show that the n-ball $B^n(a)$ is an n-manifold in R^n. **[4]**

Q3) a) Let U be an open set in R^n and $f : U \to R^n$ be of class C^r. Let $M = \{x : f(x) = 0$ and $N = \{x : f(x) \geq 0$. If M is nonempty and D $f(x)$ has rank one at each point of M, then prove that N is an n-manifold in R^n and $\partial N = M$. **[7]**

b) Define the term alternating tensor and give an example. **[5]**

c) Find length of the parametrized curve **[4]**

$\alpha(t) = (\text{ acost, asint}), 0 < t < 3\pi$.

P.T.O

Q4) a) With usual notation, prove that for any forms **[6]**

$\omega_1, \omega_2 .....\omega_k, d(d\omega_1 \wedge d\omega_2 \wedge ... \wedge d\omega_n) = 0$

b) Define the term closed form and give an example. **[5]**

c) In R^3, let ω = $xydx+2zdy-ydz$. If α(u,v) = (uv, u^2, 3u+v) then find $\alpha^* \omega$ **[5]**

Q5) a) Let M be a k-manifold in R^n and $p \in$ M. Define tangent space to M at p and show that the definition is independent of the choice of the coordinate patch at p. **[8]**

b) If $\omega = xdx+2ydy+zdz$ and $\eta = \cos zdx+ydy+\sin xdz$, then verify the identity $d(\omega \wedge \eta) = (d\omega) \wedge \eta - \omega \wedge d\eta$. **[8]**

Q6) a) It T : V$\rightarrow$W is a linear transformation, and if f and g are alternating tensors on W, then prove that $T^* (f \wedge g) = T^* f \wedge T^* g$ **[6]**

b) With usual notation, prove that if G is a symmetric tensor, then AG=0 **[5]**

c) Let $\alpha: R^3 \rightarrow R^6$ be given by $\alpha(x, y, z) = (x^2, yz, xz, y^2, xy, z^2)$. Find $d\alpha_1 \wedge d\alpha_3 \wedge d\alpha_5$. **[5]**

Q7) a) State stokes' theorem. **[4]**

b) Is the 2-sphere S^2 an orientable 2-manifold? Justify. **[4]**

c) Let A = $(0, 1)^2$. Let α : A$\rightarrow$ R^3 be given by the equation. **[8]**

α(u, v) = (u, v, u + v)

Let Y be the image set of α

Evaluate $\int_{Y_\alpha} x_1 dx_1 \wedge dx_3 + x_2 x_3 dx_1 \wedge dx_2$.

Q8) a) Give an example of a 1-manifold in R^3. **[4]**

b) Define an exact form and give an example. **[6]**

c) Give an example of a C^∞ map $\alpha: R^2 \rightarrow R^3$ such that α^{-1} is continuous but $D\alpha$ does not have rank 2 at zero. **[6]**

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Total No. of Questions : 8]

SEAT No. :

P374

[Total No. of Pages : 3

[4223]-302
M.Sc.
MATHEMATICS
MT-702: Ring Theory
(2008 Pattern) (Semester - III)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates :

1) ***Attempt any Five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that any finite integral domain is a field. **[6]**
If R is a Boolean ring as well as an integral domain them what can you say about R?

b) If R is a ring of all real valued continuous functions defined on the closed interval [0, 1] them. **[10]**

i) Prove that R is not an integral domain.
ii) Find units in R.
iii) Find nilpotent elements in R.
iv) Find idempotent elements in R.
v) Give an example of an element in R which is neither unit nor a zero-divisor in R.

Q2) a) If I and J are ideals of a ring R with $I \subseteq J$ then prove that J/I is an ideal of R/I and $\frac{R/I}{J/I} \cong \frac{R}{J}$. **[6]**

b) Show that in a commutative ring R with unity, the sum of a nilpotent element and a unit is a unit. **[4]**

c) Let R be a commutative ring with unity and $p(x) = a_0 + a_1 x + ... + a_n x^n$ be an element of R[x]. **[6]**
If a_0 is unit in R and $a_1, a_2, .., a_n$ are all nilpotent in R then prove that $p(x)$ is unit in R[x].
Find units in $Z_4[x]$.

P.T.O.

Q3) a) Prove that in a ring with unity every proper ideal is contained in a maximal ideal. **[6]**

b) If R is a commutative ring with identity 1, and each ideal in R is prime then show that R is a field. **[5]**

c) If $f(x) = x^2 + x + 1$ is an element of a ring $F_2[x]$ and $R = \dfrac{F_2[x]}{\langle f(x)\rangle}$ then show that R is a field with 4 elements. Find $\left(\overline{x+1}\right)^{-1}$. **[5]**

Q4) a) If R is an integral domain and Q is the field of fraction of R. If a field F contains a subring R' isomorphic to R then prove that the subfield of F generated by R' is isomorphic to Q. **[6]**

b) If R is a field then what is field of quotient of R. **[5]**
What happens if R = Z, 2z. Show that the field of fraction Q[x] is same as the field of fraction of Z[x]. (Q = set of all rationals).

c) If R is a commutative ring with unity 1 and I, J are ideals of R co-prime to each other then show that **[5]**

$$\frac{R}{IJ} \simeq \frac{R}{I} \times \frac{R}{J}.$$

Q5) a) Define a discrete valuation ring. Show that every discrete valuation ring is an Euclidean domain (w.r.t. a suitable norm). **[5]**

b) Show that the ring $Z\left[\sqrt{-5}\right]$ is not an Euclidean domain. **[5]**

c) i) Give an example of a PIR which is not Euclidean ring with justification. **[6]**

ii) Give an example of two elements a and b in a Euclidean domain R such that N(a) = N(b) but a and b are root not associates.

iii) Using the g.c.d. property in PID show that the ring F[x, y] is not a PID, where F is a field.

Q6) a) If I is any (non-zero) ideal in a PID R then show that **[5]**
I is prime ideal if and only if I is maximal ideal.

b) Define De-dekind-Hasse-Norm. Prove that if a commutative ring R with unity 1 has a Dedekind-Hasse Norm then R is PID. **[5]**

c) Show by an example that the quotient of a PID need not be a PID. Under what condition (s) the quotient of a PID is PID? Justify. **[6]**

Q7) a) In a unique Factorization Domain R prove that each irreducible element is a prime and hence show that the ring $Z[\sqrt{-5}]$ is not UFD. **[6]**

b) Prove that a prime number P divides on integer of the form $n^2 + 1$ if P is either 2 or is an odd prime congruent to 1 modulo 4. **[5]**

If $P \equiv 1 \bmod(4)$ is a prime in z, then show that P is not irreducible in $Z[i]$, ring of Gaussian integers.

c) If R is PID then show that there exist a multiplicative Dedekind-Hasse norm on R. **[5]**

Q8) a) If R is UFD then prove that $R[x]$ is also UFD. **[8]**

b) i) Prove that the polynomial $x^4 + 4x^3 + 6x^2 + 2x + 1$ is irreducible in $Z[x]$. **[4]**

ii) Show that the polynomial $f(x) = x$ is not irreducible polynomial in $\frac{Z}{6Z}[x]$. **[4]**

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Total No. of Questions : 8]

SEAT No. :

P375

[Total No. of Pages : 3

[4223]-303
M.Sc.
MATHEMATICS
MT-703: Mechanics
(2008 Pattern) (Semester - III)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates :

1) ***Attempt any Five questions.***
2) ***Figures to the right indicate maximum marks.***

Q1) a) Explain the concept of degrees of freedom and find the degrees of freedom for a conical pendulum. **[4]**

b) Find the force generated by the potential $\phi(x, y, z) = -x^2 y - 2z + 9$. Is this force conservative? Justify your answer. **[4]**

c) State Hamilton's principle and derive Lagrange's equations of motion, from Hamilton's principle. **[8]**

Q2) a) Derive Hamilton's equations of motion for a simple pendulum. **[6]**

b) For a projectile motion, Lagrangian $L(x, y, z, \dot{x}, \dot{y}, \dot{z}) = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$ $-mgz$, where m and g are constants. Find Lagrange's and Hamilton's equations of motion. **[6]**

c) Define virtual displacement and explain the principle of virtual work. **[4]**

Q3) a) For a 2 D harmonic oscillator, the Hamiltonian is of the form : **[4]**

$H(x, y, p_x, p_y) = \frac{1}{2m}(p_x^2 + p_y^2) + \frac{1}{2}k(x^2 + y^2)$. Show that the quantity $(xp_y - yp_x)$ is conserved.

b) Show that the following equations **[4]**

$Q = aq + bp,\ P = cq + dp$

represent a canonical transformation only when $ad - bc = 1$.

P.T.O.

c) Find the canonical transformation generated by the generating function **[4]**
$F_3(Q, p) = (e^Q - 1)^2 \tan p$.

d) Consider motion of a free particle having mass m in a plane. Express its kinetic energy in terms of plane polar coordinates and their time derivatives. **[4]**

Q4) a) Show that the following transformation is canonical by finding the Poisson brackets: **[5]**

$$Q = \sqrt{e^{-2q} - p^2},\ P = \cos^{-1}(pe^q).$$

b) Show that if F and G are two constants of motion then their Poisson brackets [F,G] is also a constant of motion. **[6]**

c) Prove that $\frac{dH}{dt} = \frac{\partial H}{dt}$, where H is the Hamiltonian function. **[5]**

Q5) a) Find the stationary function of the integral **[4]**

$$\int_{-1}^{1} ((y')^2 - 2xy)\, dx,\ y(-1) = -1, y(1) = 1.$$

b) If the equations of transformation do not depend explicitly on time and if the potential energy is velocity independent, then show that H is the total energy of the system. **[6]**

c) Using cylindrical coordinates (ρ, ϕ, z) write the Hamilton's equations of motion for a particle of mass m moving in a force field of potential $V(\rho, \phi, z)$. **[6]**

Q6) a) Explain the active and passive view of coordinate transformations. **[4]**

b) State and prove rotation formula. **[6]**

c) Define infinitesimal rotations. Show that the matrix representing infinitesimal rotations is antisymmetric. **[6]**

Q7) a) Define orthogonal transformations in 3-dimensions. **[3]**

b) State and prove Euler's theorem on the motion of a rigid body. **[7]**

c) Derive the matrix of transformation in terms of the Euler angles: (ϕ, θ, ψ). **[6]**

Q8) a) Define central force motion. Show that it is always planar. Further show that the areal velocity is constant. **[5]**

b) Derive the following differential equation for the path of the particle in the central force field **[6]**

$$\frac{d^2u}{d\theta^2} + u = -\frac{f\left(\frac{1}{u}\right)}{mh^2u^2}$$

c) State and prove Kepler's second law of planetary motion. **[5]**

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Total No. of Questions : 8]

SEAT No :

P376

[Total No. of Pages : 4

[4223] - 304

M.Sc.

MATHEMATICS

MT - 704 : Measure and Integration

(2008 Pattern) (Sem. - III)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates :

1) *Attempt any five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Define following terms with suitable example. **[6]**

i) σ-algebra.

ii) Outer Measure.

iii) Measurable function.

b) If $E_i \in B$ with $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that **[6]**

$$\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n .$$

c) Show that $\mu(E_1 \Delta E_2) = 0$ implies $\mu E_1 = \mu E_2$ provided that $E_1, E_2 \in B$. **[4]**

Q2) a) Suppose that to each α in a dense set D of real numbers there is assigned a set $B_\alpha \in B$ such that $B_\alpha \subset B_\beta$ for $\alpha < \beta$. Then show that there exist a unique measurable extended real valued function f on X such that $f \leq \alpha$ on B_α and $f \geq \alpha$ on $X \sim B_\alpha$. **[6]**

b) Let $< f_n >$ be a sequence of measurable functions that converge to a function f except at the points of set E of measure zero. Then prove that f is a measurable function if μ is complete. **[6]**

P.T.O.

c) Show that the set of numbers in [0, 1] which possess decimal expansions not containing the digit 5 has measure zero. **[4]**

Q3) a) State and prove Fatou's Lemma. **[6]**

b) If f and g are nonnegative measurable functions and a and b are nonnegative constants, then show that $\int af + bg = a\int f + b\int g$. **[6]**

c) Show that if f is a non negative measurable function, then $f = 0$ a.e. if and only if $\int f dx = 0$. **[4]**

Q4) a) Let (X, B) be a measurable space, $< \mu_n >$ a sequence of measures that converge set wise to a measure μ and, $f_n >$ a sequence of nonnegative measurable functions that converge pointwise to the function f then show that $\int f d\mu \leq \underline{\lim} \int f_n d\mu_n$. **[8]**

b) i) Define signed measure.

ii) Let v be a signed measure on (X, B) and $E \in B$ with $v(E) > 0$. Then show that there exist A, a set positive with respect to v, such that $A \subset E$ and $v(A) > 0$. **[8]**

Q5) a) Let v be a signed measure on the measurable space (X, B) then prove that there is a positive set A and a negative set B such that $X = A \cup B$ and $A \cap B = \phi$. **[8]**

b) Show that the following conditions on the signed measure μ and v on (X, B) are equivalent. **[8]**

i) $v << \mu$.

ii) $|v| << |\mu|$.

iii) $v^+ << \mu$.

iv) $v^- << \mu$.

Q6) a) Let F be a bounded linear functional on $L^p(\mu)$ with $1 \leq p < \infty$ and μ a σ-finite measure. Then show that there is a unique element g in L^q where $1/p + 1/q = 1$, such that $F(f) = \int fgd\mu$ with $\|F\| = \|g\|q$. **[6]**

b) Show that the class B of μ^* - measurable sets is a σ-algebra. **[6]**

c) Let μ be a measure on an algebra G, μ^* the outer measure induced by μ, and E any set. Show that for $\varepsilon > 0$, there is a set $A \in$ G with $E \subset A$ and $\mu^*(A) \leq \mu^*(E) + \varepsilon$. **[4]**

Q7) a) i) Define product measure.

ii) Let E (subset of $X \times Y$) a set in $R_{\sigma\delta}$ and x be a point of X. Then show that E_x (x cross section E) is a measurable subset of Y. **[8]**

b) Let E be a set in $R_{\sigma\delta}$ with $\mu \times v(E) < \infty$. Then show that the function g defined by $g(x) = \mu E_x$ is a measurable function of x and $\int gd\mu = \mu \times v(E)$. **[8]**

Q8) a) Let (X, G, μ) and (Y, B, v) be two complete measure spaces and f an integrable function on $X \times Y$. Then prove the following : **[8]**

i) For almost all x the function f_x defined by $f_x(y) = f(x, y)$ is an integrable function Y.

ii) For almost all y the function f_x defined by $f^y(x) = f(x, y)$ is an integrable function X.

iii) $\int_y f(x, y)\, dv(y)$ is an integrable function on X.

iv) $\int_X f(x, y)\, d\mu(y)$ is an integrable function on Y.

b) Let μ be a measure on an algebra G and μ^* the outer measure induced by μ. Then prove that the restriction $\overline{\mu}$ of μ^* to the μ^* - measurable sets is an extension of μ to σ-algebra containing G. **[8]**

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Total No. of Questions : 8] **[Total No. of Pages : 3**

P376

[4223] - 304

M.Sc.

MATHEMATICS

MT - 704 : Mathematical Methods - I

(Sem. - III) (Old)

Time : 3 Hours] ***[Max. Marks : 80***

Instructions to the candidates :

1) *Attempt any five questions.*

2) *Figures to the right indicate full marks.*

Q1) a) Find whether the following series converges or diverges. **[6]**

$$\sum_{n=1}^{\infty}\frac{1}{n!}=1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}+\ldots$$

b) Find the interval of convergence of the power series. **[5]**

$$\sum_{n=1}^{\infty}\frac{x^{2n-1}}{(2n-1)!}$$

c) Find the series for $(x + 1) \sin x$. **[5]**

Q2) a) Given $f(x)=\begin{cases}1, & -\pi<x<0\\ 0, & 0<x<\pi\end{cases}$

Expand $f(x)$ in an exponential fourier series of period 2π . **[8]**

b) Explain comparison test, integral test, ratio test for convergence of series of positive terms. **[8]**

P.T.O.

Q3) a) Solve the boundary value problem

$$y_{tt}(x, t) = a^2 y_{xx}(x, t),\ 0 < x < L,\ t > 0$$

Subject to conditions $y(0, t) = 0;\ y(1, t) = 0$

$y_t(x, t) = 0;\ y(x, 0) = f(x),\ 0 \le x \le L.$ **[8]**

b) Define even and odd function. Also sketch the graph of the functions $f(x) = x^2$ and $f(x) = \cos x$, give geometric interpretation of the graphs of function. **[8]**

Q4) a) Prove that $\int_{-1}^{1} p_m(x)\, p_n(x)\, dx = 0$ if $m \neq n$. **[8]**

b) Express the following integrals as Beta function and evaluate it. **[8]**

i) $J = \int_0^1 \frac{x^4}{\sqrt{1-x^2}}\, dx$

ii) $I = \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}}$

Q5) a) Prove that $\int_0^{\infty} \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$. **[8]**

b) i) Define : error function.

ii) Show that $\operatorname{erf}(-x) = -\operatorname{erf}(x)$.

iii) Show that $\operatorname{erf}(\infty) = 1$. **[8]**

Q6) a) Obtain the Rodrigues formula for the Lagurre polynomial $L_n^{(\alpha)}(x)$. **[8]**

b) Prove that, $\sqrt{(p)}\ \sqrt{(1-p)}=\dfrac{\pi}{\sin\pi p}$. **[8]**

Q7) a) Use Laplace transformation to solve the differential equation. **[8]**

$y''' + 2y'' - y' - 2y = 0,\ y(0) = y'(0) = 0$ and $y''(0) = 6$

b) Find the value of $L^{-1}\left\{\dfrac{s}{\left(s^2+a^2\right)^2}\right\}$. **[4]**

c) Obtain $L\{\sin\sqrt{t}\}$. **[4]**

Q8) a) Show that $\dfrac{d^s}{dx^s}H_n(x)=\dfrac{2^n\,n!H_{n-s}(x)}{(n-s)!}$ for $x<n$. **[8]**

b) State and prove convolution theorem for Fourier transform. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P377

[Total No. of Pages : 2

[4223]-305
M.Sc.
MATHEMATICS
MT-705: Graph Theory
(2008 Pattern) (Sem. - III)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates :

1) ***Attempt any Five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that the isomorphism relation is an equivalence relation on the set of simple graphs. **[6]**

b) How many simple graphs are there on a fixed set of six vertices? Draw all non-isomorphic simple graphs on a set of four vertices. **[5]**

c) Show that the Petersen graph has girth 5. **[5]**

Q2) a) Prove that an edge is a cut-edge if and only if it belongs to no cycle. **[8]**

b) Compute the diameter and radius of the complete bipartite graph $K_{m,n}$. **[2]**

c) Prove that in an even graph, every non-extendible trail is closed. **[6]**

Q3) a) Prove that a graph is bipartite if it has no odd cycle. **[8]**

b) Show that the k-dimensional cube Q_k is k-regular bipartite graph. **[5]**

c) State and prove the Handshaking lemma. **[3]**

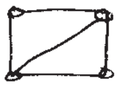
Q4) a) State and prove the Havel-Hakimi theorem. **[10]**

b) For an n-vertex graph G with ($n \geq 1$), prove that G is connected and has no cycles if and only if G has no loops and has, for each $u,v \in V(G)$, exactly one u,v-path. **[6]**

Q5) a) Prove that the center of a tree is a vertex or an edge. **[8]**

b) i) State the Matrix Tree Theorem.

ii) Explain the Matrix Tree Theorem for the following graph.

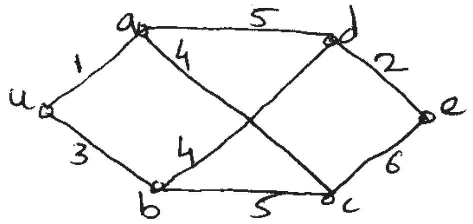


[8]

P.T.O.

Q6) a) Prove that in a connected weighted graph G, Kruskal's algorithm constructs a minimum-weight spanning tree. **[8]**

b) Using Dijkstra's algorithm find shortest distance from u to every other vertex in the following graph. **[5]**



c) Find the perfect matchings in complete graph K_n. **[3]**

Q7) a) Prove that a matching M in a graph G is a maximum matching in G if and only if G has no M-augmenting path. **[8]**

b) Explain the Augmenting Path Algorithm to produce a maximum matching. **[5]**

c) i) Define the Harary graphs. **[3]**

ii) Draw $H_{5,8}$ and $H_{5,9}$.

Q8) a) Prove that if G is a simple graph, then $k(G) \leq k'(G) \leq \delta(G)$. **[8]**

b) Prove that if G is 2-connected, then the graph G' obtained by dividing an edge of G is 2-connected. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P378

[Total No. of Pages : 4

[4223]-306
M.Sc.
MATHEMATICS
MT-706 : Numerical Analysis
(2008 Pattern) (Semester - III)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates :

1) *Attempt any Five questions.*
2) *Figures to the right indicate full marks.*
3) *Use of non-programmable, scientific calculator is allowed.*

Q1) a) Assume that g is a continuous function and that $\{p_n\}_{n=1}^{\infty}$ is a sequence generated by fixed-point iterations. If $\lim_{n\to\infty} p_n = p$, then prove that p is a fixed point of $g(x)$. **[6]**

b) Let $g(x) = x^2 + x - 4$. Can fixed-point iteration $p_n = g(p_{n-1})$ be used to find the solution to the equation $x = g(x)$? Why? **[5]**

c) Start with the interval $[a_0, b_0]$ and use the bisection method to find an interval of width 0.05 that contains a solution of the equation $\exp(x) - x - 2 = 0$, $[a_0, b_0] = [1.0, 1.8]$. **[5]**

Q2) a) Assume that $A>0$ is a real number and let $p_0>0$ be an initial approximation to $\sqrt{A}$. Define the sequence $\{p_k\}_{k=0}^{\infty}$ using the recursive rule

$$p_k = \frac{p_{k-1} + A/p_{k-1}}{2} \text{ for } k = 1,2,...$$

Prove that the sequence $\{p_k\}_{k=0}^{\infty}$ converges to $\sqrt{A}$. **[5]**

b) Let $f(x) = x^3 - 3x - 2$. **[6]**

i) Find the Newton-Raphson formula $g(p_{k-1})$.

ii) Start with $p_0 = 2.1$ and compute p_1, p_2, p_3 and p_4.

iii) Is the sequence converging quadratically or linearly?

P.T.O.

c) Solve LY = B, UX = Y and verify that B = AX for $B^T = (12, 18, 8, 8)$, where A = LU is **[5]**

$$\begin{bmatrix} 2 & 4 & -4 & 0 \\ 1 & 5 & -5 & -3 \\ 2 & 3 & 1 & 3 \\ 1 & 4 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 1 & -\frac{1}{3} & 1 & 0 \\ \frac{1}{2} & \frac{2}{3} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & -4 & 0 \\ 0 & 3 & -3 & -3 \\ 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

Q3) a) Assume that $f \in C^{N+1}[a,b]$ and that $x_0, x_1, ..., x_N \in [a,b]$ are N+1 nodes. If $x \in [a,b]$, then prove that

$$f(x) = P_N(x) + E_N(x)$$

where $P_N(x)$ is a polynomial that can be used to approximate $f(x)$:

$$f(x) \approx P_N(x) = \sum_{k=0}^{N} f(x_k) L_{N,k}(x)$$

The error term $E_N(x)$ has the form

$$E_N(x) = \frac{(x-x_0)(x-x_1)........(x-x_N)}{(N+1)!} f^{N+1}(c),$$

for some value $c = c(x)$ that lies in the intervat $[a,b]$. **[6]**

b) Compute the divided difference table for the tabulated function $f(x) = \frac{3.6}{x}$.

x :	1	2	3	4	5
$f(x)$:	3.60	1.80	1.20	0.90	0.72

Write down the Newton's polynomial $P_4(x)$. **[5]**

c) Let $f(x) = exp(x)$. Calculate approximation to $f'(2.3)$ with $h = 0.01$, and compare with $f'(2.3) = exp(2.3)$. **[5]**

Q4) a) Derive the formula. **[5]**

$$f''(x_0) \approx \frac{2f_0 - 5f_1 + 4f_2 - f_3}{h^2}.$$

b) Explain Gauss-Seidel iteration method for solving a system of n equations in n unknowns. **[6]**

c) Use the Jacobian matrix to find the differential changes (du, dv, dw) when the independent changes from (1, 3, 2) to (1.02, 2.97, 2.01) for the system. **[5]**

$$u = f_1(x, y, z) = x^3 - y^3 + y - z^4 + z^2,$$
$$v = f_2(x, y, z) = xy + yz + xz$$
$$w = f_3(x, y, z) = \frac{y}{xz}.$$

Q5) a) Find the number M and the step size h so that the error $E_s(f, h)$ for the composite Simpson rule is less than 5×10^{-9} for the approximation

$$\int_2^7 \frac{dx}{x} \approx S(f, h).$$ **[5]**

b) Consider a general interval [a, b]. Show that Simpson's rule produces exact results for the functions $f(x) = x^3$ and $f(x) = x^2$. **[6]**

c) Verify that Boole's rule ($M = 1, h = 1$) is exact for polynomials of degree ≤ 5 of the form $f(x) = c_5x^5 + c_4x^4 + \ldots + c_1x + c_0$, over [0, 4]. **[5]**

Q6) a) Use Euler's method to solve the I.V.P.

$$y' = \frac{t - y}{2} \text{ on } [0, 3] \text{ with } y(0) = 1.$$

Compare solutions for $h = 1, \frac{1}{2}, \frac{1}{4}, \text{and } \frac{1}{8}$. **[8]**

b) Use $h = 0.05$ and Euler's method to find $\{x_1, y_1\}$ and $\{x_2, y_2\}$. Solve the system $x' = 2x + 3y$, $y' = 2x + y$ with the initial condition $x(0) = -2.7$ and $y(0) = 2.8$ over the interval $0 \leq t \leq 1.0$. **[8]**

Q7) a) Find the eigen pairs λ_j, V_j for the matrix.

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

Also, show that the eigenvectors are linearly independent. **[8]**

b) If X and Y are vectors with the same norm, then prove that there exists an orthogonal matrix P such that

$$Y = PX,$$

where $P = I - 2\,WW^T$

and $$W = \frac{X - Y}{\|X - Y\|_2}.$$ **[8]**

Q8) a) Find the error and relative error in the following case. **[3]**

$x = 3.141592$ and $\bar{x} = 3.14$.

b) Compare the results of calculating $f(500)$ and $g(500)$ using six digits and rounding. The functions are $f(x) = x\left[\sqrt{x+1} - \sqrt{x}\right]$ and

$$g(x) = \frac{x}{\sqrt{x+1} + \sqrt{x}}.$$ **[5]**

c) Consider the non-linear system.

$f_1(x, y) = x^2 + y^2 - 2 = 0,$

$f_2(x,y) = xy - 1 = 0.$.

i) Verify that the solutions are $(1, 1)$ and $(-1, -1)$.

ii) What difficulties might arise if we try to use Newton's method to find the solutions? **[5]**

d) Consider $f(x) = 2 + \sin(2\sqrt{x})$. Use the trapezoidal rule with 11 sample points to compute an approximation to the integral of $f(x)$ taken over $[1, 6]$. **[3]**

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Total No. of Questions : 8]

SEAT No. :

P379

[Total No. of Pages : 2

[4223] - 401
M.Sc.
MATHEMATICS
MT - 801 : Field Theory
(2008 Pattern) (Semester - IV)

Time : 3 Hours] *[Max. Marks : 80*

Instructions to the candidates :

1) ***Attempt any five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Prove that every non-constant polynomial $f(x) \in F[x]$ has a root in some extension field E of F. **[8]**

b) Show that $f(x) = x^3 + x + 2$ reducible over Z_7. **[4]**

c) Find the basis of $Q\left(\sqrt{3}, \sqrt[3]{2}\right)$ over $Q\left(\sqrt{3}\right)$. **[4]**

Q2) a) Let $p(x)$ be an irreducible polynomial in $F[x]$ and u be a root of $f(x)$ in an extension E of F. Then prove that $F(u)$, the subfield of E generated by F and u is the set.

$\{b_0 + b_1u + ... + b_mu^m \,/\, b_0 + b_1x + ... + b_mx^m \in F[x]\}$.

Further show that if degree of $p(x)$ is n, then $\{1, u, u^2, ----, u^{n-1}\}$ forms a basis of $F(u)$ over F. **[8]**

b) Find the splitting field E of $x^4 - 1 \in Q[x]$ over Q. Also find $[E : Q]$. **[3]**

c) Let $E = Q\left(\sqrt[3]{2}\right)$, and $\theta = \sqrt[3]{2}$, then find the inverse of $1 + \theta$ in E. **[5]**

Q3) a) Define normal extension and illustrate it by an example. **[4]**

b) If $f(x) \in F[x]$ is irreducible over F, then prove that all roots of $f(x)$ have the same multiplicity. **[8]**

c) Construct a field with 4 elements. **[4]**

Q4) a) Let E be a finite extension field of a field F. Then prove that if $E = F(\alpha)$ for some α in E then there are only finite number of intermediate fields between F and E. **[6]**

b) Let $F = Q\left(\sqrt{2}\right)$ and $E = Q\left(\sqrt[4]{2}\right)$. Is E normal extension of F? Is E normal extension of Q? Justify your answer. **[6]**

P.T.O.

c) Does there exists a field with 742 elements? Justify your answer. **[4]**

Q5) a) Let E be an extension of a field F. Define the group of F-automorphisms of E. Illustrate your definition by an example. **[5]**

b) Let E be a finite separable extension of a field F. If E is normal extension of F, then prove that F is the fixed field of G (E/F). **[6]**

c) Show that the group $G\,(Q\,(\alpha)/\mathrm{Q})$, where $\alpha^5 = 1$ and $\alpha \neq 1$, is isomorphic to the cyclic group of order 4. **[5]**

Q6) a) Let E be a Galois extension of F, K be any subfield of E containing F. Prove that K is normal extension of F if and only if $G\,(E/K)$ is normal subgroup of $G\,(E/F)$. **[8]**

b) Find the Galois group $G\,(K/Q)$, where $K = Q\left(\sqrt{3}, \sqrt{5}\right)$. Find all subgroups of $G\,(K/Q)$ and their corresponding fixed fields. **[8]**

Q7) a) Let E be the splitting field of $x^n - a \in F\,[x]$. Then prove that $G\,(E/F)$ is solvable group. **[8]**

b) Write a note on squaring a circle. **[6]**

c) What is the problem of duplicating a cube. **[2]**

Q8) a) Let $E = Q\left(\sqrt[3]{2}, w\right)$ be an extension of a field Q, where $w^3 = 1$, $w \neq 1$. Find E_H, where $H = \{\sigma_0, \sigma_1\}$, where σ_0 is the identify automorphism of E and $\sigma_1\left(\sqrt[3]{2}\right) = \sqrt[3]{2}\,w$, $\sigma_1\,(w) = w^2$. **[6]**

b) Prove that every finite extension of a finite field is normal extension. **[5]**

c) Find the minimum polynomial of $\sqrt{-1+\sqrt{2}}$ over Q. **[5]**

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Total No. of Questions : 8]

SEAT No. :

P382

[Total No. of Pages : 2

[4223]-404
M.Sc.
MATHEMATICS
MT-804: Algebraic Topology
(2008 Pattern) (Semester - IV)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates :

1) *Attempt any Five questions.*
2) *Figures to the right indicate full marks.*

Q1) a) Prove that the homotopy relation is an equivalence relation. **[6]**

b) Let f and g be homotopic mappings of X into Y and h be a continuous mapping of Y into Z. Prove that hf and hg are homotopic mappings of X into Z. **[5]**

c) Let $f_0 : X \to Y$ is homotopic to $f_1 : X \to Y$ and $g_0 : Y \to Z$ is homotopic to $g_1 : Y \to Z$. Show that $g_0 f_0 : X \to Z$ is homotopic to $g_1 f_1 : X \to Z$. **[5]**

Q2) a) Prove that the relation of being of the same homotopy type is an equivalence relation. **[6]**

b) Prove that the closed unit ball B^3 in R^3 is contractible but the sphere S^2 in R^3 is not contractible. **[5]**

c) Let $A \subset B \subset X$. Suppose B is a retract of X and A is a retract of B. Show that A is a retract of X. **[5]**

Q3) a) Prove that a non-empty open connected subset of R^n is path connected. **[6]**

b) Let f and g be paths in X such that $f * \overline{g}$ exists and is a closed path. Then $f * \overline{g}$ is homotopic to a null path if and only if f is homotopic to g. **[5]**

c) Let $A \subset X$ be a path connected subset and $\{A_n : n \in Z^+\}$ a collection of path connected subsets of X each of which intersects with A. Show that $A \cup \{\cup_n A_n\}$ is path connected. **[5]**

P.T.O.

Q4) a) Let $f: X \to Y$ be a continuous map. Prove that there exists a homomorphism $f^* : \pi_1(X, x_0) \to \pi_1(Y, f(x_0))$, where x_0 is any point of X. **[6]**

b) Let X and Y be of same homotopy type and $\phi : X \to Y$ be a homotopy equivalence. Prove that $\phi^* : \pi_1(X, x) \to \pi_1(Y, \phi(x))$ is an isomorphism for any $x \in X$. **[5]**

c) If A is a strong deformation retract of X, show that the inclusion map $i : A \to X$ induces an isomorphism $i^* : \pi_1(A, a) \to \pi_1(X, a)$ for any point $a \in A$. **[5]**

Q5) a) Using the theory of homotopy relation and lifting lemma, prove that every non-constant complex polynomial has a root. **[6]**

b) Prove that the circle S^1 is not a retract of the disc B^2. **[5]**

c) Find the fundamental groups of $R^2 - \{0, 0\}$ and the sphere S^2. **[5]**

Q6) a) Show that a covering map is an open map. **[6]**

b) Let $p : \tilde{X} \to X$ be a covering map and $X_0 \subset X$. Let $\tilde{X}_0 = p^{-1}(X_0)$. Show that $p_0 : \tilde{X}_0 \to X_0$ is a covering map, where $p_0(x) = p(x)$. **[5]**

c) Let $p : \tilde{X} \to X$ be a covering map and let $f_1, f_2 : \tilde{X} \to X$ be two liftings of $f : Y \to X$. Suppose Y is connected and there exists $y_0 \in Y$ such that $f_1(y_0) = f_2(y_0)$. Then prove that $f_1 = f_2$. **[5]**

Q7) a) Define a fibration. Give an example of a fibration. Prove that if $p: E \to B$ is a fibration, then any path f in B with $f(0) \in p(E)$ can be lifted to a path in E. **[6]**

b) Prove that the composite of fibrations with unique path lifting is a fibration with unique path lifting. **[5]**

c) Suppose $p : E \to B$ has unique path lifting. Prove that p has path lifting property for path connected spaces. **[5]**

Q8) a) Prove that the diameter of a p-simplex $s_p = (a_0, a_1, ..., a_p)$ is the length of its longest edge. **[6]**

b) Using topological dimension, prove that R^n is homeomorphic to R^m if and only if $n = m$. **[5]**

c) Prove that two different complexes may have the same polyhedron. **[5]**

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Total No. of Questions : 8]

P382 **[Total No. of Pages : 2**

[4223]-404

M.Sc.

MATHEMATICS

MT-804 : Mathematical Methods - II

(Semester - IV) (Old)

Time :3 Hours] ***[Max. Marks :80***

Instructions to the candidates :

1) ***Attempt any Five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Show that the function $u(x)=\sin(\pi x/2)$ is a solution of the Fredholm integral equation $u(x)-\frac{\pi^2}{2}\int_0^1 \mathrm{k}(x,t)u(t)\,dt=\frac{X}{2}$ **[6]**

b) Form the integral equation corresponding to the differential equation $y''-2xy=0$ with initial conditions

$y(0)=\frac{1}{2},\ y'(0)=y''(0)=1.$ **[6]**

c) Define : **[4]**

i) Voltera integral equation of first kind.

ii) Separable Kernel.

Q2) a) Reduce the following boundary value problem into an integral equation

$\frac{d^2u}{ds^2}+\lambda u=0$ with $u(0) = 0$, $u(l) = 0$. **[8]**

b) Show that, the eigen functions of a symmetric Kernel corresponding to different eigen values are orthogonal. **[8]**

Q3) a) Find the eigenvalues and eigen functions of the homogeneous integral equation $\phi(x)=\lambda\int_0^\pi(\cos^2\times\cos 2t+\cos 3\times\cos^3 t)\phi(t)dt.$ **[8]**

b) Solve the homogeneous Fredholm integral equation $\phi(s)=\lambda\int_0^1 e^s e^t\phi(t)\,dt.$

[8]

P.T.O.

Q4) a) Prove that, if a Kernel is symmetric then all its iterated Kernels are also symmetric. **[8]**

b) Find the iterated Kernels for the Kernel $K(s,t) = \sin(s-2t),\ 0 \le s,t \le 2\pi$. **[8]**

Q5) a) Find the Neumann series for the solution of the integral equation **[10]**

$$u(x) = 1 + \int_0^x xtu(t)\,dt$$

b) Solve $y(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2}\, y(t)\,dt$ by resolvent Kernel. **[6]**

Q6) a) Prove that $\frac{d}{dX}\frac{\partial F}{\partial y^1} - \frac{\partial F}{\partial y} = 0$ (Euler - Lagrange's equation) with usual notations. **[8]**

b) Find the extremals of the functional $I = \int_{x_0}^{x_1} (y'^2 / x^3)\,dx$. **[8]**

Q7) a) State and prove isoperimetric problem. **[8]**

b) Show that the curve which extremizes the functional

$I = \int_0^{\pi/4} (y''^2 - y^2 + x^2)\,dx$ under the conditions $y(0) = 0, y'(0) = 1,$ **[8]**

$y(\pi/4) = y'(\pi/4) = 1/\sqrt{2}$ is $y = \sin x$.

Q8) a) State and prove principle of least action. **[8]**

b) State and prove Harr theorem. **[8]**

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Total No. of Questions : 8]

SEAT No. :

P383

[Total No. of Pages : 2

[4223]-405

M.Sc.

MATHEMATICS

MT-805: Lattice Theory

(2008 Pattern) (Semester - IV)

Time :3 Hours] *[Max. Marks :80*

Instructions to the candidates :

1) ***Attempt any Five questions.***
2) ***Figures to the right indicate full marks.***

Q1) a) Let N_0 be the set of all non-negative integers. Define $m \leq n$ if and only if there exists $k \in N_0$ such that $n = km$ (that is m divides n). Prove that N_0 is a lattice under this relation. **[5]**

b) Does there exists a lattice without infinite chains but is not of finite length? Justify your answer. **[5]**

c) Define a congruence relation on a lattice L and prove that the set of all congruence relations on L forms a lattice. **[6]**

Q2) a) Let L be a finite distributive lattice. Prove that L is pseudocomplemented. Is finiteness necessary to prove the assertion. Justify your answer. **[5]**

b) Prove that a lattice L is distributive if and only if for any two ideals I, J of L, $I \vee J = \{i \vee j \mid i \in I, j \in J\}$. **[6]**

c) Let P be a prime ideal of a lattice L. Prove that P satisfies the following condition (*). **[5]**

(*): For a, b, $c \in L$, if a, $b \wedge c \in P$ implies $(a \vee b) \wedge (a \vee c) \in P$.

Is the condition (*) implies the primeness of P? Justify your answer.

Q3) a) Let L be a pseudocomplemented lattice. Prove that $S(L) = \{a^* / a \in L\}$ is a bounded lattice. **[8]**

b) Prove that a lattice is modular if and only if it does not contain a pentagon (N_5) as a sublattice. **[8]**

P.T.O.

Q4) a) Prove that in a finite lattice L, the following statement (S) is true. **[5]**

(S): For a, $b \in L$ with $a \nleq b$, there exists a join-irreducible element $j \in L$ such that $j \leq a$ and $j \nleq b$.

b) Show that the following inequalities hold in any lattice. **[6]**

i) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee z)$;

ii) $(x \wedge y) \vee (x \wedge z) \leq x \wedge (y \vee (x \wedge z))$.

c) Let I and J be ideals of a distributive lattice L. If $I \wedge J$ and $I \vee J$ are principal ideals then prove that I and J are also principal ideals. **[5]**

Q5) a) Prove that a lattice is distributive if and only if it is isomorphic to ring of sets. **[8]**

b) Let L be a bounded distributive lattice with $|L| > 1$ and the set $P(L)$ of all prime ideals of L is unordered. Prove that L is complemented. **[8]**

Q6) a) Prove that every prime ideal of a Boolean lattice is maximal and conversely. **[8]**

b) Let L be a distributive lattice, let I be an ideal, let D be a dual ideal of L, and let $I \cap D = \theta$. Then prove that there exists a prime ideal P of L such that $P \supseteq I$ and $P \cap D = \theta$. **[8]**

Q7) a) Prove that every modular lattice satisfies the upper and the lower covering conditions. **[4]**

b) State and prove fixed point theorem for lattices. **[6]**

c) Define conditionally complete lattice and illustrate with an example. Prove that every conditionally complete lattice is complete, if it has the least and greatest element. **[6]**

Q8) a) Prove that every chain is a distributive lattice. **[4]**

b) Let $\phi : L \rightarrow L_1$ be a onto homomorphism. Then prove that the image of an atom is an atom. **[4]**

c) Prove that every prime ideal is a meet-irreducible element of a ideal lattice but not conversely. **[6]**

d) Prove that every finite distributive lattice is semimodular. **[2]**

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